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Nonlinear dynamics of an elastically coupled *N* hybrid micro-cantilever array subjected to electrostatic excitation

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Extended Abstract

In recent years nonlinear dynamics of micro- and nano- beam arrays have been extensively studied as they offers significantly increased sensitivity of the frequency bandwidth in comparison to the frequency bandwidth from a single element. In Ref. [1] arrays of four and eight resonant cantilevers were fabricated using polysilicon and electrically excited for high sensitive mass detection. A numerical method to manipulate intrinsic localized mode (ILM) was discussed for a coupled micorcantilever array by determining a nonlinear coupling coefficient of coexisting and dynamical stability of standing ILMs [2]. Periodic, quasiperiodic and chaotic dynamics of an initial boundary value problem for a three element array with several internal resonances was investigated in Ref. [3]. A stepped cantilever composed of a bottom-up silicon nanowire to a top-down silicon microcantilever subjected to electrostatic excitation was analysed theoretically and experimentally which facilitates an improvement in mass sensing resolution with respect to that of the microcantilever standalone [4]. In Ref. [5] a mechanically coupled two beam cantilever array was analysed to show enhanced sensitivity of an AFM array as compared to an individual beam. In this work, we numerically study the nonlinear dynamics of *N* elastically coupled hybrid micro-cantilever beams to investigate various coexisting periodic and aperiodic solutions.

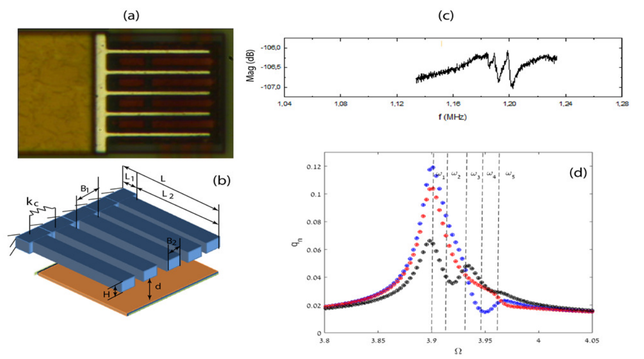

Figure 1: (a) SEM image (b) Definition sketch (c) Experimental measurements (d) Numerical frequency response (with Q=200, V_{ac}=0.707 V, V_{dc}=30 V) for an array of five micro-cantilever beams.

Table 1: Comparison of frequency ratios between experimental measurements and Numerical analysis.

Frequency ratios	Experimental	Numerical
f_5/f_4	1.0084	1.0084
f_5/f_3	1.018	1.018
f_4/f_3	1.01	1.01

Problem Formulation and Validation

We derive a continuum mechanics-based nonlinear model for an array of *N* micro-cantilever beams with step-like heterogeneity of its width subjected to electrostatic excitation along out-of-plane direction as shown in Fig. 1 (a) and (b). Each beam element in coupled array is divided into two fields: the cantilever with width B_2 and a base with width B_1 as shown in Fig. 1 (b). The base elements are assumed to be rigidly fixed at the fixed end and bonded to the base of their respective cantilever sections. Each base element is coupled to its neighboring base elements through an elastic spring constant k_c [5], which acts in the z direction only. Neglecting nonlinearity in base field, the two field equations of motion [4] governing an array of *N* micro-cantilever beams augmented with gyroscopic and bending nonlinearity, nonlinear damping are derived. We employ Galerkin method and obtain modal dynamic equations. Subsequently, we perform equilibrium analysis for an array of five microbeams to find the effective value of elastic spring constant yielding the theoretical linear frequencies which satisfies the experimental measurements as shown in Fig. 1(c). Finally, with the clue of linear analysis we perform forced frequency analysis numerically (Fig. 1(d)) to find an effective elastic spring constant to yield theoretical frequency ratios which are in good agreement with the experimental measurements as shown in Table 1.

Numerical Analysis of *N* micro-cantilever beams

A numerical analysis of five beam array and twenty five beam array is shown in Fig. 2. The frequency response curve for an array of five beams (Fig. 2(a)) reveals four bifurcation regions (Fig, 2(b)). It is observed that the regions I, II (small amplitude) and IV have periodic solutions whereas regions II (large amplitude) and III have quasiperiodic solutions. The time history and phase plane with Poincare/ points in region III are shown in Fig. 2 (c) and (d). The frequency response curve for an array of twenty five beams (Fig. 2(e)) depicts four bifurcation regions (Fig. 2(f)). The regions I and IV have periodic solutions. The region II for small amplitude is quasiperiodic. The region II for large amplitude and the subregion A of region III have chaotic solutions. The chaotic solutions transforms into the complex quasiperiodic solutions in subregion B and the complicated quasiperiodic solutions ends as simple tori in subregion C of region III. The time history and Poincare/ map for chaotic solution in region III are shown in Fig. 2 (g) and (h).

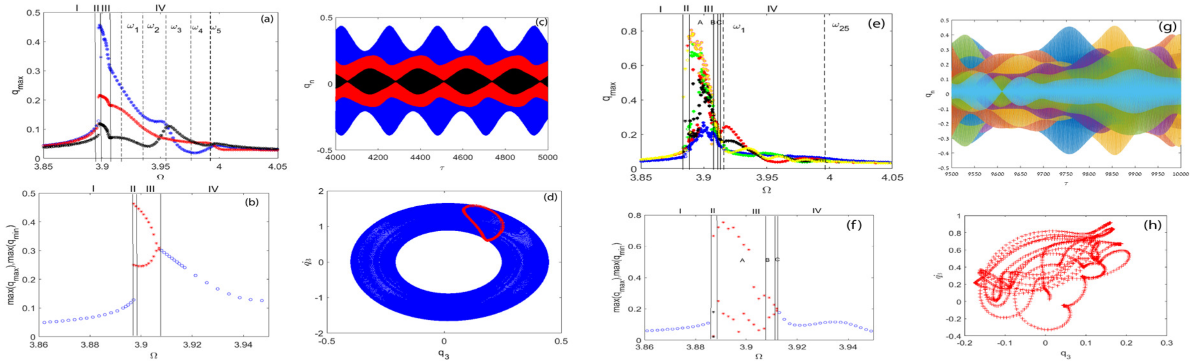

Figure 2: **.** (a) Frequency response (b) Bifurcation diagram showing different solutions for region I-IV (c) Time history (d) Phase plane with Poincare/ points (Ω =3.9, 4600$\leq \tau \leq$5000, 249 points) for an array of five beams (N=5) with Q=300, V_{ac}=0.707 V and V_{dc}=30 V. In (a) and (c) blue, red and black indictes q_3, $q_{2,4}$, $q_{1,2}$ amplitudes. In (b) blue circles are periodic solutions and red dots are quasiperiodic solutions. (e) Frequency response (f) Bifurcation diagram showing different solutions for region I-IV (g) Time history (h) Poincare/ map (Ω =3.887, 8000$\leq \tau \leq$10000, 1238 points) for an array of twenty five beams (N=25) with Q=300, V_{ac}=0.9 V, V_{dc}=30 V. In (e) and (g) blue, red, green, black and yellow colors correspond to different q_n for n=1,4,6,9,13 respectively. In (f) blue circles are periodic solutions and red dots are aperiodic solutions. Circle, asterisk represents forward and backward numerical sweeps.

Conclusions

This study provides a mathematical model to study the nonlinear dynamics of an array of *N* elastically coupled hybrid micro-cantilever beams with step-like heterogeneity of its width under the influence of gyroscopic and bending nonlinearity, nonlinear damping subjected to electrostatic excitation. The bifurcation structures of five and twenty five beam arrays reveals coexisting periodic solutions, quasiperiodic energy transfer between coupled beams and chaotic solutions which is novel application to enhance the sensing capability significantly.

References

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