Uncertainty: Models and Measures

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APERIODIC WHIRLING OF NONLINEAR STRINGS: CHAOTIC DYNAMICS OR MODELLING UNCERTAINTY

Oded Gottlieb

In this study we investigate the onset and evolution of aperiodic whirling of a nonlinear string and compare the characteristics of chaotic whirling obtained in a deterministic model to system response including model uncertainty. The uncertainty is incorporated in the model by introducing a weak stochastic perturbation to the generalized forces describing dissipation and external excitation. Results demonstrating modified threshold values for the onset of aperiodic whirling may further bridge the documented discrepancy between approximate theoretical predictions and experimental results.

1 Introduction

Whirling or out-of-plane response of parametrically excited taut strings and slack elastic cables has been the subject of renewed interest recently. While both theoretical and experimental analysis demonstrate existence of complex periodic and aperiodic responses, only qualitative agreement has been found between theoretical predictions and experimental observations (cf. Perkins, 1992; O'Reilly & Holmes, 1992; Benedettini & Rega, 1994; Nayfeh et al.,1995). The theoretical investigation of string and cable dynamics typically includes approximate analysis of the (slow) amplitude equations derived by perturbation techniques(cf. Miles 1984) complemented by their numerical simulation (cf. Johnson & Bajaj,1989) and consists mainly of fundamental mode dominance. This enables reduction of the space-time boundary value problem into a finite differential state space where nonlinear interactions can be analyzed by classical (cf. Nayfeh & Mook, 1979) and modern (cf. Guckenheimer & Holmes, 1983) techniques. The stability analysis results in an approximate bifurcation structure of the string system which can then be complemented by numerical analysis of the complete space-time dynamics (cf. Rubin & Gottlieb, 1996) and ultimately by experiments (cf. Molteno & Tufillaro, 1990; Lee & Perkins, 1994; Nayfeh et al., 1995). The discrepancy between theoretical and measured results appear in the threshold values representing the onset of periodic and aperiodic whirling and in the crisis representing whirling collapse and return to periodic motion (Bajaj & Johnson, 1994).

Steady state periodic whirling is characterized by a periodic angular momentum which remains of a single sign whereas aperiodic whirling (quasiperiodic, chaotic) can be observed by an amplitude modulated angular momentum which can alternate sign corresponding to a change in the whirl direction. Furthermore, an unpredictable change in the amplitude modulated whirl direction can be observed when the whirling response of a deterministic model becomes chaotic. The thresholds for periodic and quasiperiodic whirling are explained via asymptotic theory by local bifurcation analysis of the constructed weakly nonlinear slowly varying amplitude evolution equations (Miles, 1984). The onset of periodic whirling is realized by the emergence of nonplanar fixed points in the evolution equations and the onset of quasiperiodic whirling is described by the further emergence of limit cycles in evolution space via the mechanism of a Hopf bifurcation. Furthermore, existence of chaotic motion was demonstrated numerically for both evolution equations and amplitude modulated solutions (Johnson & Bajaj, 1989) and explained theoretically via global bifurcations of the limiting slowly varying evolution system (O'Reily & Holmes, 1992; O'Reily,1993).

Some of the possible explanations offered for the discrepancy between theory, simulation and experiment consist of the arbitrary selection of periodic boundary excitation under the fundamental mode assumption, lack of inclusion of higher modes and in the modelling of internal (material damping) external (aeroelastic drag) and boundary dissipation mechanisms. In the reported studies the only dissipation mechanism incorporated was that of assumed (or measured) linear modal damping. The linear damping values were typically obtained by logarithmic decrement analysis of free vibration decay that were found to be generally very small. We note that while calibration of damping coefficients is typically obtained via small amplitude free vibration, the whirling amplitudes can be of much larger magnitude particularly near the crisis describing the return bifurcation from whirling to periodic motion. Thus modeling of large amplitude nonlinear dissipation mechanisms is essential for investigation of the reported aperiodic phenomena. Recently, the discrepancy reported by O'Reilly and Holmes (1992) was verified by a numerical study (Rubin and Gottlieb, 1996) where the simulations of the complete string problem indicate that the forcing amplitude threshold is about five times smaller than that observed in experiments. In order to raise the threshold, additional (linear) equivalent damping and a material nonlinearity were included. However, while the reported experimental periodic onset values were obtained, both model modifications did not reveal aperiodic response. In an additional study (Gottlieb, 1996), quadratic aeroelastic damping (Hsu, 1975) and a Coulomb law of friction and its continuous function approximation (Feeny & Moon, 1989) were incorporated to the fundamental string model. We note that although these nonlinear damping forms did not appear in the 'pluck' test calibration of the experiment, their inclusion revealed the appearance of both quasiperiodic and chaotic whirling.

An additional explanation for the reported discrepancy can be that of a modeling uncertainty which governs the unpredictable nature of the aperiodic taut string response. This uncertainty can be realized as an apparent random fluctuation of various system parameters which contain a wide spectrum of frequencies. Consequently, a nonlinear model incorporating deterministic or stochastic parameter (time varying) changes is required to investigate the possible wide banded spectral outcome describing both random or chaotic responses. This can be achieved by various means including the incorporation of a nonstationary process describing system excitation (cf. Neal & Nayfeh, 1990) or by modification of the harmonic filed or boundary excitation by a weak random perturbation (cf. Roberts, 1985; Pascual & Vazquez, 1985). We note that investigation of nonlinear stochastic dynamical systems exhibiting multiple coexisting solutions, reveal that incorporation of noise will enhance complexity of the response (Frey & Simiu, 1993) and change the system bifurcation structure. Furthermore, it has been shown that chaotic dynamics can be annihilated by various forms of nonstationarity (Moslehy & Evan-Iwanowsky, 1991) and that the topological structure (eg. circular or fractal Poincar map corresponding to quasiperioidc or chaotic dynamics) of an aperiodic attractor in a nonlinear deterministic system without noise can be distorted beyond recognition after incorporation of noise. Consequently, selection of a modelling uncertainty should preserve the measured amplitude modulated structure if available from experiment.

In this study we utilize the Lagrangian formulation to derive the fundamental transverse and longitudinal modes and include linear modal damping and quadratic aeroelastic drag as generalized forces. We incorporate uncertainty in the dissipation and forcing model via weak stochastic perturbations generated by an added white Gaussian noise. The excitation considered is $\mathcal{O}(\varepsilon)$ so that the primary resonance, damping and nonlinearity balance each other. Results demonstrating the modified threshold values for whirling onset are obtained and compared to those based on a classical modal damping assumption. The apparent amplitude modulated response due to stochastic variability describing modelling uncertainty is analyzed and compared to that obtained via the deterministic dynamical system exhibiting chaotic whirling.

2 Equations of Motion for the Fundamental Mode

The equations of motion for the taut string can be obtained from the Lagrangian density describing the difference between kinetic and strain energies for a geometrically nonlinear configuration:

$$\mathcal{L}(s,t) = \int_0^L \left[\frac{\rho}{2}\left(u_t^2 + v_t^2 + w_t^2\right) - \frac{EA}{2}(a^* - 1)^2 \right] ds \tag{1}$$

where s is the arclength coordinate at time t; $u(s,t)$, $v(s,t)$ and $w(s,t)$ denote the longitudinal, vertical and horizontal displacements respectively; $a^*(s,t)$ is the stretch and ρ, E, A are the mass density per unit length, Young's modulus and cross-sectional area respectively.

The position vector $r^*(s,t)$ incorporates the prestrech of the string in the longitudinal direction where a is a constant ($a \geq 1$) determining the magnitude of prestretching as $r^* = ve_1 + we_2 + (u + as)e_3$ where $a = 1 + T_0/EA$ and T_0 is the initial tension of the string.

The boundary conditions considered in this work include harmonic excitation of the ends in the vertical direction: $u(0,t) = u(L,t) = w(0,t) = w(L,t) = 0$, $v(0,t) = B_0 \cos \Omega t$, $v(L,t) = B_L \cos \Omega t$.

In order to obtain a fundamental mode representation from the Lagrangian, the following assumptions are required: i) small displacements and strains enabling simplification of the strain energy in (1) via a Taylor series expansion, ii) an assumed spatial mode representation incorporating the time dependent boundary conditions of (5). We first recall the comprehensive analysis of Narasimha (1968) demonstrating that the influence of the longitudinal response is $u = O(v^2,\ w^2)$ (Nayfeh et al, 1995). Consequently, assuming a simple harmonic form for $(v(s),\ w(s))$ results in a spatially doubled harmonic representation for $u(s)$.

The stretch of the string $a^*(s,t)$ is defined in terms of the magnitude of the tangent vector: $a^* = |\partial r^*/\partial s|$. Thus, We propose the following assumed mode form for the displacement vector:

$$r^* = \left[A_1(t)\sin\frac{\pi s}{L} + f(s,t)\right] e_1 + \left[A_2(t)\sin\frac{\pi s}{L}\right] e_2 + \left[A_3(t)\sin\frac{2\pi s}{L} + as\right] e_3 \qquad (2)$$

where $A_i (i = 1,2,3)$ are time dependent amplitudes and $f(s,t)$ corresponds to the contribution of boundary excitation in the vertical direction.

We note that the spatially doubled harmonic representation for the longitudinal displacement $(u(s))$ has been verified numerically (Rubin & Gottlieb, 1996). However, as noted by O'Reilly & Holmes (1992), there are no guidelines for the section of $f(s,t)$ in (2) other than its compliance with the boundary conditions resulting in a large number of candidate functions. This arbitrariness in problem formulation requires validation of results with an independent experiment. We select the following simple assumed form for $f(s,t)$ and demonstrate that the resulting whirling solutions are similar to those obtained in an equivalent numerical experiment:

$$f(s,t) = \left[B_0 + (B_L - B_0)\left(\frac{s}{L}\right)\right]\sin\Omega t \qquad (3)$$

Substitution of (2) and (3) into (1) and integration over the domain including terms to $O(A_i^3, A_iB_j^2)$ (where $i = 1,2,3; j = 0, L$) results in the following equations for the fundamental mode (Gottlieb, 1996):

$$\begin{pmatrix} \ddot{A}_1 \\ \ddot{A}_2 \end{pmatrix} + \omega_0^2 \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} + \left(\tfrac{\pi\omega_1}{aL}\right)^2 \left[\left(\tfrac{3}{8a}\right)(A_1^2 + A_2^2) + \left(\tfrac{L}{\pi}\right)A_3 + \begin{pmatrix} 3 \\ 1 \end{pmatrix} \tfrac{G^2}{2\pi^2 a}\right] \times$$
$$\times \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} \frac{2}{\pi}\Omega^2 F(t) \\ 0 \end{pmatrix} \qquad (4)$$
$$\ddot{A}_3 + (2\omega_1)^2 A_3 + \left(\tfrac{\pi\omega_1}{aL}\right)^2 \left[\left(\tfrac{L}{2\pi}\right)(A_1^2 + A_2^2) - \tfrac{2}{a}\left(A_1^2 + A_2^2 + \tfrac{2G^2}{\pi^2}\right)A_3\right] = 0$$

where $F_{1,2} = (B_L + - B_0)\sin(\Omega t)$ and the linear natural frequencies for the transverse and longitudinal modes are $\omega_0 = (1 - 1/a)^{0.5}\omega_1$, $\omega_1 = \pi(EA/\rho)^{0.5/L}$.

Recall that the fundamental mode for the longitudinal direction was assumed to have a double spatial harmonic due to $u = O(v^2, w^2)$. This result stems from the assumption that the natural frequency of the longitudinal mode (ω_1) is much larger than that of the transverse mode (ω_0) which in turn implies that the longitudinal modes are not excited. Consequently, the longitudinal inertia and small terms to $O(A^3)$ in (4) can be neglected resulting in two coupled nonlinear hardening Duffing equations for A_1 and A_2. Note that as the stretch constant (a) is very close to unity, the limiting valued for the nonlinear stiffness coefficient in the reduced system (Gottlieb, 1996) is shown to be identical to that derived by other multiple scale analysis (Miles, 1984; Nayfeh et al, 1995).

We introduce damping to the fundamental Hamiltonian modal subsystem (4) as a dissipation function incorporating a linear modal and quadratic damping (Gottlieb,

1996) and incorporate the model uncertainty via an additive perturbation to the forcing function ($F(t)$) and allow the damping coefficients to vary.

$$D(\dot{A}_i) = \frac{1}{\rho}\left[c_1\dot{A}_i + c_2\dot{A}_i\left|\dot{A}_i\right|\right] \quad , \quad c_1 = \bar{c}_1 + \sigma \tag{5}$$

where the noise selected is from a Gaussian controlled by the standard deviation ($\sigma \ll 1$).

3 The Influence of Variable Nonlinear Damping and Forcing

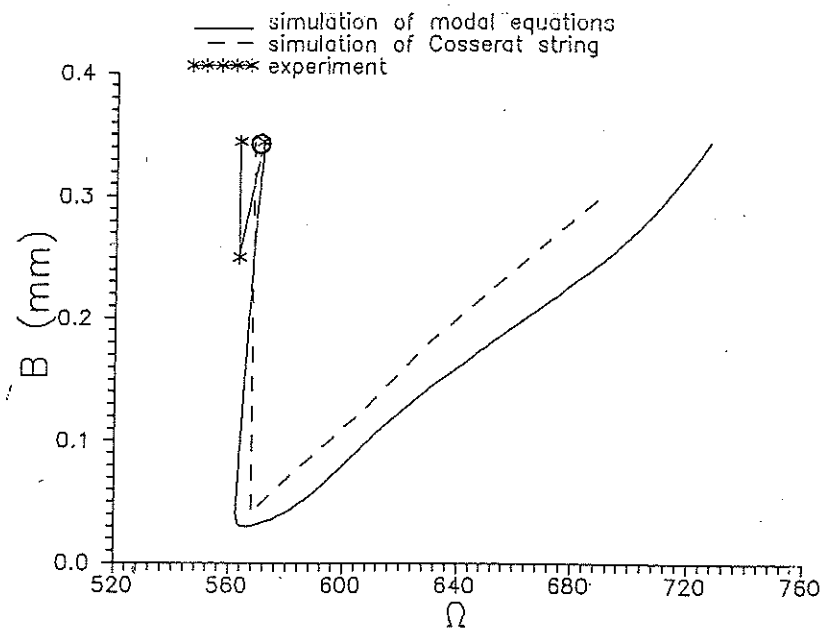
The fundamental influence of the nonlinear damping mechanisms can be demonstrated for the case of small amplitude response in the slowly varying evolution equations derived from the fundamental equations of motion. After rescaling [$A = \zeta R$, where $\zeta = (a-1)L$ and $\tau = \Omega t$] and applying a generalized averaging procedure we obtain the following:

$$\begin{aligned} \dot{R}_1 &= \tfrac{\epsilon}{2}\left[-\left(\gamma_1 + \tfrac{8\gamma_2}{3\pi}R_1\right)R_1 - \tfrac{1}{4}R_1R_2^2\sin 2(\phi_2-\phi_1) - \mu\cos\phi_1\right] \\ R_1\dot{\phi}_1 &= \tfrac{\epsilon}{2}\left[-\alpha R_1 + \tfrac{3}{4}R_1\left(R_1^2+R_2^2\right) - \tfrac{1}{2}R_1R_2^2\sin^2(\phi_2-\phi_1) + \mu\sin\phi_1\right] \\ \dot{R}_2 &= \tfrac{\epsilon}{2}\left[-\left(\gamma_1 + \tfrac{8\gamma_2}{3\pi}R_2\right)R_2 + \tfrac{1}{4}R_1^2R_2\sin 2(\phi_2-\phi_1)\right] \\ R_2\dot{\phi}_2 &= \tfrac{\epsilon}{2}\left[-\alpha R_2 + \tfrac{3}{4}R_2\left(R_1^2+R_2^2\right) - \tfrac{1}{2}R_1^2R_2\sin^2(\phi_2-\phi_1)\right] \end{aligned} \tag{6}$$

where the small parameter is $\epsilon^{0.5} = \beta\zeta/\Omega$, α is a detuning [$\epsilon\alpha = 1-(\omega_0/\Omega)^2$], μ is the forcing [$\epsilon\mu = 2(B_L + B_0)/\pi\zeta$]. Furthermore (6) reduces ($\sigma = 0$) to the classical dynamical system derived from the reduced system for a weakly nonlinear string with linear modal damping (Gottlieb, 1996). Following Miles (1984), solution of the equilibrium system obtained from (6) will yield the thresholds and bifurcation curves for periodic and quasiperiodic whirling corresponding to pitchfork and Hopf bifurcations respectively. Furthermore, whirling motion is bounded by a saddle-node bifurcation governing the crisis mechanism controlling return to planar system response ($R_2 = 0$).

The analytically obtained bifurcation structure does not address the discrepancies reported between theory and experiment(O'Reilly and Holmes, 1992; Perkins, 1992; Nayfeh et al, 1995) where the chaotic whirling response occurs for finite excitation (in our notation epsilon $\mu = 0.636$ for parameters in O'Reily and Holmes, 1992). This can be observed in the stability diagram (Figure 1a) obtained from numerical simulation of the system (4) where the damping mechanism (5) is linear ($c_2 = 0$). Similar results have been shown in the numerical investigation of the string via the theory of Cosserat(Rubin and Gottlieb, 1996).

Following the Hopf bifurcation results obtained from analysis of (6) we simulate the dynamical system (4, 5) without noise to yield a quasiperiodic phase plane ($x = A_1 + 0.5B\sin\Omega t, y = A_2$) and Poincare' map (Figures 2a,b). However, inclusion of weak noise in the damping mechanism reveals that the quasiperiodic response for the system without noise has evolved to the apparent form of a chaotic attractor with weak noise (Figure.3a,b). Furthermore, while results from numerical simulation



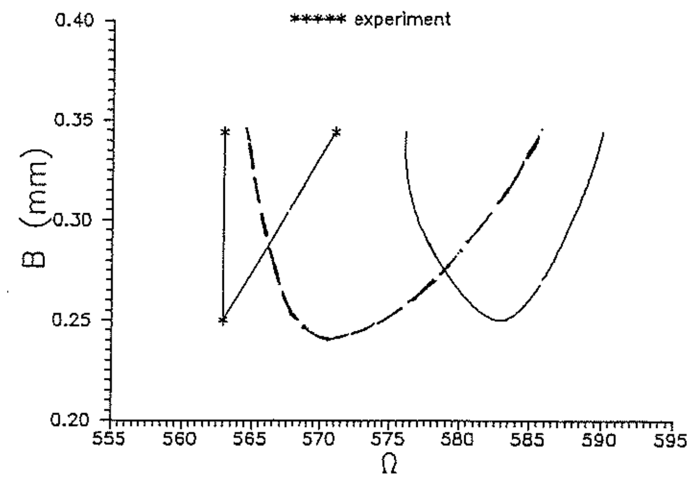


Figure 1: Stability diagram: a) Linear damping model without noise (top). b) non-linear damping model with and without weak noise (bottom).

of (4, 5) without noise reveal that the modified damping mechanism (5) reproduces the stability threshold of periodic whirling (solid line in Figure 1b), inclusion of the nonstationary variations widen the domain of stability (dashed line in Figure 1b).

The influence of variable nonlinear damping consists of modification of the fundamental threshold of periodic whirling and reveals and enables the appearance of documented aperiodic solutions in the form of a noisy chaotic attractor.

In closing we note again that nonlinear damping mechanisms have not been identified in the reported literature of aperiodic whirling and only linear damping was extracted from free vibration 'pluck' tests. Furthermore, the reports of noisy additive components are typically not considered as they fall into the region of instrumentation sensitivity. However, due to the large amplitude response of periodic and aperiodic whirl (particularly near crisis) and the difficulties in identification of dissipation mechanisms during nonlinear forced vibration, the influence of damping and forcing parameter variability should be considered. We note that without the inclusion of an added model uncertainty in the form of noise the threshold of aperiodic whirling revealed only quasiperiodic response whereas inclusion of noise enhanced the bifurcation structure resulting in the perturbation of the quasiperioidc solution to an apparent 'noisy' chaotic attractor. Future research incorporating stochastic bifurcation analysis (Ariaratnam, 1994) from both a dynamical systems approach (Arnold,1988) or identification of a change in the probability distribution of the response (Horsthemke & Lefever, 1984) may also yield further insight and result in alternative bifurcation thresholds.

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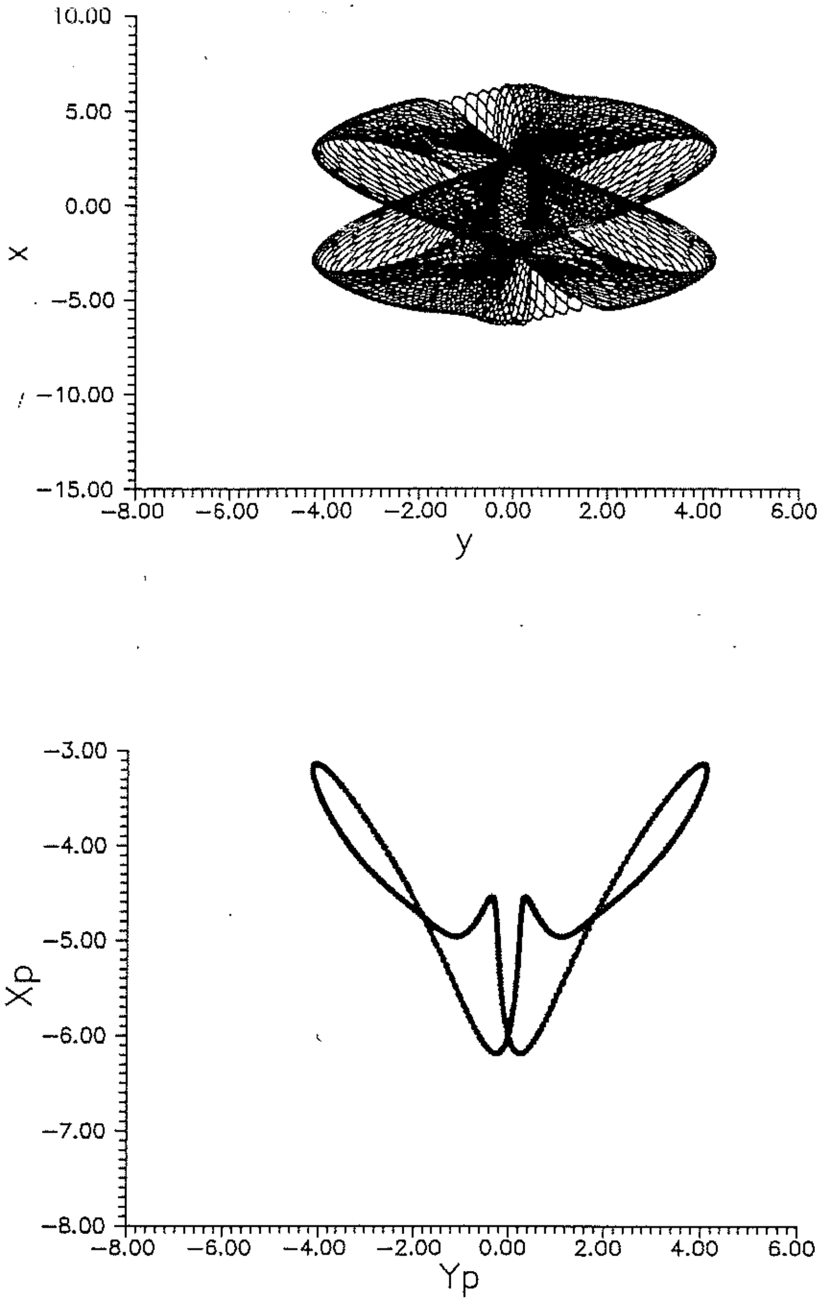

Figure 2: Quasiperiodic response of fundamental system without noise: a) phase plane (top). b) Poincare' map (bottom).

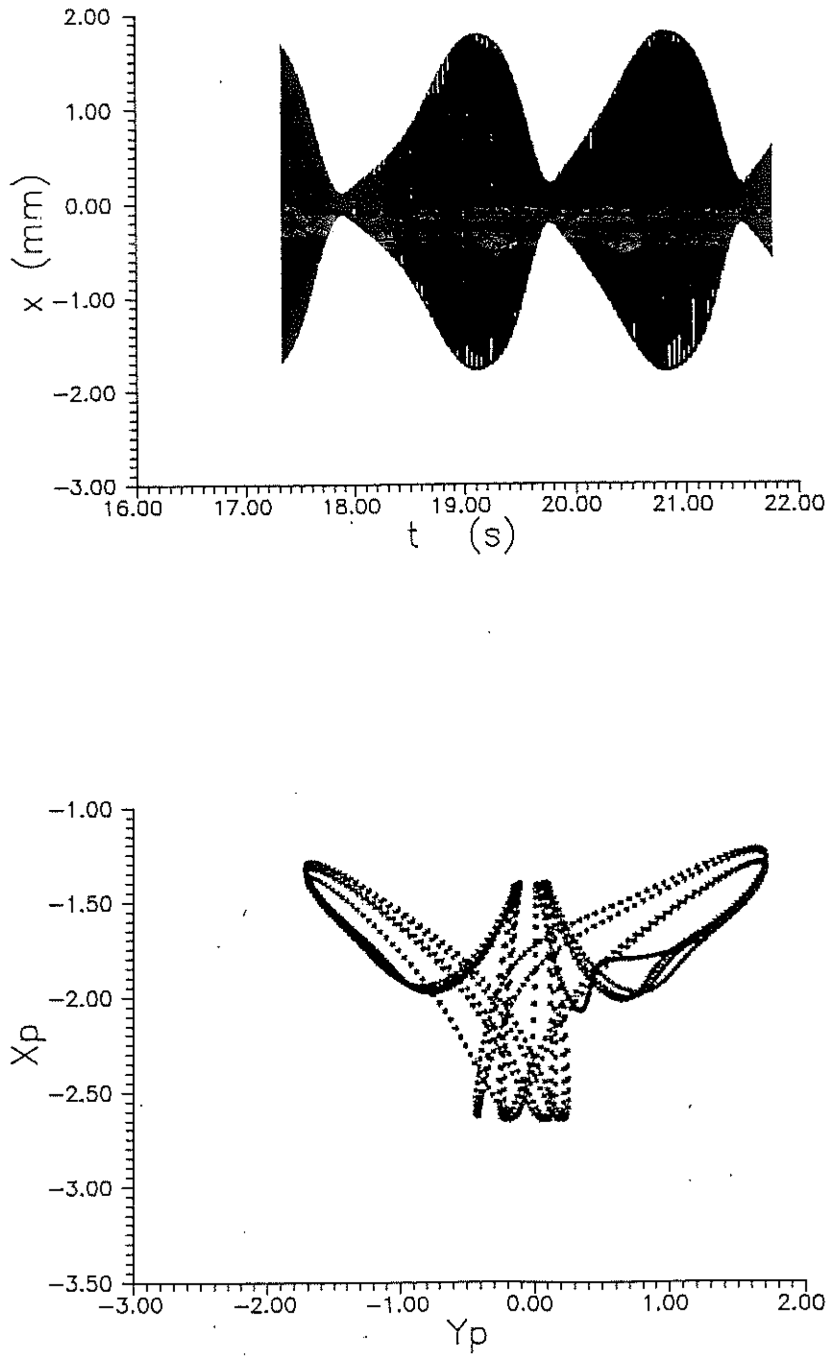

Figure 3: Aperiodic response of modified system with weak noise: a) Time series plane (top). b) Poincare' map (bottom).

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