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ABSTRACT

The behavior of smart localized structural elements for nonlinear vibration control of a taut string is investigated in this manuscript. A nonlinear lumped mass dynamical system is derived and analyzed numerically to reveal conditions for possible forced vibration reduction. The strategy employed consists of an open loop excitation approach enabled via actuation of a smart element by a slight harmonic change of its length. Results of a bifurcation analysis reveal possible vibration reduction via two distinct mechanisms: i) parametric excitation that enables reduction of external forcing and ii) energy transfer from the directly excited vertical response to both rotation and horizontal motions.

Keywords: nonlinear vibration, smart localized control, taut string, internal resonances, quasiperiodic dynamics.

1. INTRODUCTION

Smart structural activation via piezoelectric patches or layers for identification and control of undesired vibrations has been successfully implemented in several applications[1,2]. Examples include flexible elements in longitudinal[3] and bending vibrations[4]. Recently, several examples have been introduced in the domain of microelectromechanical systems where applications typically include thin film coatings of piezoelectric layers[5]. Examples are microcantilevers for scanning probe microscopy[6,7] and in applications to surgery[8]. Control of vibrations using smart structures with optimally placed distributed elements[9] has been implemented for vibration absorption[10] using both concepts of linear[11] and nonlinear[12,13] control.

The nonlinear dynamic response of slack cables and taut strings is of great interest in a variety of technical problems including vibrations of offshore mooring and towing systems, transmission line conductors, and connecting micro and nanowires. While these examples differ in the form of environmental field loads, they incorporate similar boundary conditions and internal mechanical forces. Furthermore, under various time dependent environmental conditions, these systems exhibit finite amplitude response that includes periodic (ultrasubharmonics, mode locking) and aperiodic (quasiperiodic and chaotic dynamics) behavior[14-17]. Of particular interest are the internal resonance dynamics that transfer energy from an excited mode to its (coupled) counterpart in both cables and strings[18-20].

Control of cable vibrations has been implemented by passive dampers placed near the supports or active boundary control using linear and nonlinear control schemes[21,22]. However, instabilities can occur due to coupling of the mechanical nonlinearities and the control feedback[23]. Indeed, planar harmonic boundary actuation of a taut string in transverse[24] or both transverse and longitudinal directions[25] induces aperiodic, out-of-plane whirling near primary resonance. We note that passive vibration absorption[26] and open loop resonance cancellation control[27,28] have been successful in reducing motion induced by based excitation of a pendulum.

To date, control of cables and strings has not been done by smart elements in the field. Thus we are motivated to investigate the application of control via smart localized elements in the span of a taut string system that is subject to forced vibration in the vertical plane. We implement an open loop strategy in the form of a small periodic perturbation of the smart element length and investigate possible reduction of system response.

This paper includes derivation of a planar lumped mass taut string dynamical system which incorporates a localized piezoelectric sleeve that can change its length (periodically) to the aim of controlling externally excited vibrations in the vertical plane. Analysis of the system without the element actuation yields finite amplitude dynamics including coexisting solutions (jump phenomena), ultra- subharmonics, internal resonances, quasiperiodic and chaotic response. We focus here on a fundamental period-three response. Inclusion of the element actuation is considered as a periodic perturbation of the element length with frequency of vibration that can coincide or is incommensurate with that of the

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forcing frequency. Results from a numerical bifurcation analysis are discussed to reveal possible vibration reduction via both parametric excitation and energy transfer from the directly excited vertical response to both rotation and horizontal motions.

2. DYNAMICAL SYSTEM

The taut string is described here by a planar lumped mass model connected via two pretensioned springs (with stiffness k and initial length ℓ_0) to a vibrating frame of (width 2b) that oscillates harmonically (V(t)). The model incorporates a small localized element which consists of a rigid body piezoelectric sleeve (of length 2a) located at midspan. The mass of the system (m) describes both the string and the localized sleeve. The length of the element (2a) can vary in a controlled manner and includes a constant part and a time varying part $(a = a_0 + A(t))$. Thus, the system mass remains constant whereas the element moment of inertia (I(t)) is a quadratic function of time. The generalized coordinates selected are the translation $(X(t), Y(t))$ from the reference midspan position at rest and the angular rotation of the element $(\theta(t))$.

We construct the dynamical system utilizing a Lagrangian approach. The Lagrangian (L) is assembled from the following kinetic (KE) and potential (PE) energies.

$$KE = \frac{m}{2}[\dot{X}^2 + (\dot{Y} + \dot{V})^2] + \frac{I}{2}\dot{\theta}^2 \quad , \quad PE = \frac{k}{2}[(\Delta_1 - \ell_0)^2 + (\Delta_2 - \ell_0)^2] \tag{1}$$

where the time dependent inertia and stretch are:

$$I = \frac{m}{12}(2a)^2 \quad , \quad \Delta_{1,2} = [(b \pm X - a\cos\theta)^2 + (Y \pm a\sin\theta)^2]^{1/2} \tag{2}$$

and the localized time dependent element length and input excitation of the frame are:

$$a = a_0 + A(t) \quad , \quad A(t) = a_1 \sin\Omega_U t \quad ; \quad V(t) = V_1 \sin\Omega_V t \tag{3}$$

In this research we assume simple linear damping so that the generalized damping forces can be obtained from a Rayleigh dissipation function.

$$G = \frac{1}{2}(c_1\dot{X}^2 + c_2\dot{Y}^2 + c_3\dot{\theta}^2) \tag{4}$$

The equations of motion are obtained by substitution of the system energies (1) and the dissipation function (4) into Lagrange's equations $(j = 1,2,3)$.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \left(\frac{\partial L}{\partial q_j}\right) = Q_j \quad ; \quad L = (KE - PE) \quad , \quad Q_j = -\frac{\partial G}{\partial \dot{q}_j} \quad , \quad q_j = (X, Y, \theta)^T \tag{5}$$

We scale the system coordinates and displacements (X,Y,U,V) by half the span (b) and scale time by the square root of the ratio between the string stiffness (k) and the system mass (m).

$$x = \frac{X}{b} \quad , \quad y = \frac{Y}{b} \quad , \quad u = \frac{a}{b} \quad , \quad v = \frac{V}{b} \quad , \quad \tau = \omega t \quad ; \quad \omega^2 = \frac{k}{m} \tag{6}$$

Consequently, after rearrangement of terms, the equations of motion (5) are obtained in nondimensional form to yield the model dynamical system.

$$x'' = -\left[2x - \frac{\mu}{\Delta_1}(1 + x - u\cos\theta) + \frac{\mu}{\Delta_2}(1 - x - u\cos\theta)\right] - \delta_1 x'$$

$$y'' = -\left[2y - \frac{\mu}{\Delta_1}(y - u\sin\theta) - \frac{\mu}{\Delta_2}(y + u\sin\theta)\right] - \delta_2 y' - v'' \qquad (7)$$

$$\theta'' = -\left(\frac{3}{u}\right)\left[2\sin\theta - \frac{\mu}{\Delta_1}((1+x)\sin\theta - y\cos\theta) - \frac{\mu}{\Delta_2}((1-x)\sin\theta + y\cos\theta)\right] - \frac{\delta_3\theta'}{u^2} - 2\theta'\left(\frac{u'}{u}\right)$$

where the prime denotes $d/d\tau$ and the nondimensional stretch is:

$$\Delta_{1,2} = [(1 \pm x - u\cos\theta)^2 + (y \pm u\sin\theta)^2]^{1/2} \qquad (8)$$

and the nondimensional element length ($u(\tau)$) and frame acceleration ($v''(\tau)$) are:

$$u(\tau) = \alpha + \tilde{u}(\tau) \quad , \quad \tilde{u}(\tau) = \beta\sin\Omega_2\tau \quad ; \quad v''(\tau) = -\gamma\Omega_1^2\sin\Omega_1\tau \qquad (9)$$

The nondimensional system parameters are:

$$\alpha = \frac{a_o}{b}, \quad \beta = \frac{a_1}{b}, \quad \gamma = \frac{V_1}{b}, \quad \delta_{1,2} = \frac{c_{1,2}}{m\omega}, \quad \delta_3 = \frac{3c_3}{mb^2\omega}, \quad \mu = \frac{\ell_0}{b}, \quad \Omega_{1,2} = \frac{\Omega_{V,U}}{\omega} \qquad (10)$$

We note that the dynamical system (7) is not singular as the stretch (Δ_{12}) and time dependent element length (u) are always positive as $\alpha > \beta$. Furthermore, as the element length is smaller than the frame width, $\alpha < 1\,(a_0 < b)$, it can be shown that the initial string length parameter is small, $\mu \le (1-\alpha)$, as the initial taut spring length is smaller than the frame half width $(\ell_0 \le b - a_0)$.
The dynamical system (7) can be reduced to the following single degree of freedom equation for symmetric motion without rotation $(x = \theta = 0)$.

$$y'' = -2y[1 - \frac{\mu}{\sqrt{(1-u)^2 + y^2}}] - \delta_2 y' - v'' \qquad (11)$$

This limiting equation, expanded to third order assuming $O(y) = O(\tilde{u}(\tau))$, is a hardening Duffing equation subject to combined external $(v''(\tau))$ and parametric excitation $(\tilde{u}(\tau))$ [29].

$$y'' + 2[1 - (\frac{\mu}{1-\alpha})]y + [\frac{\mu}{(1-\alpha)^3}]y^3 + \delta_2 y' = [\frac{2\mu}{(1-\alpha)^2}][1 - \frac{3y^2}{(1-\alpha)^4}]\tilde{u}y - v'' \qquad (12)$$

We note that this limiting equation without parametric excitation $(\tilde{u}(\tau) = 0)$ reduces (for $\mu = (1-\alpha)$) to the strongly nonlinear Duffing equation (e.g. without linear stiffness) investigated by Ueda[30,31] to yield a rich ultrasubharmonic bifurcation structure including several types of chaotic strange attractors.

In order to evaluate the coupling between the generalized coordinates we expand the system (7) to cubic order:

$$x'' = -\{2x + [\frac{2\mu u}{(1-u)^2}]y\theta - [\frac{2\mu}{(1-u)^3}](xy^2 + u^2 x\theta^2\} - \delta_1 x'$$

$$y'' = -\{2y[1-(\frac{\mu}{1-u})] - [\frac{2\mu}{(1-u)^2}]x\theta + [\frac{\mu}{(1-u)^3}][y(y^2 - 2x^2) + u(1+2u)y\theta^2]\} - \delta_2 y' - v''$$

$$\theta'' = -(\frac{3}{u})\{2\theta[1-(\frac{\mu}{1-u})] - [\frac{2\mu}{(1-u)^2}]xy - \frac{1}{3}\theta^3[1 - \frac{\mu(1+u+u^2)}{(1-u)^3} + [\frac{\mu}{(1-u)^3}]\cdot[-(2u)x^2\theta + (1+2u)y^2\theta]\} - \delta_3 \frac{\theta'}{u^2} - 2(\frac{u'}{u})\theta' \quad (13)$$

The truncated system reveals both quadratic and cubic coordinate coupling and consists of a nonlinear coupled system with possible internal resonances that is subject to single and multiple frequency excitation[32].

3. NATURAL FREQUENCIES AND INTERNAL RESONANCES

The natural frequencies of the dynamical system (7) are easily observed from the truncation (13).

$$\omega_x = \sqrt{2} \quad , \quad \omega_y = \sqrt{2[1-(\frac{\mu}{1-\alpha})]} \quad , \quad \omega_\theta = \sqrt{\frac{3}{\alpha}}\,\omega_y \quad (14)$$

The smallest (and fundamental) frequency is ω_y and we distinguish between three possible cases: i) $\omega_y < \omega_x < \omega_\theta$, ii) $\omega_y < \omega_x = \omega_\theta$, iii) $\omega_y < \omega_\theta < \omega_x$.

The conditions for existence for each case are a function of the nondimensional element length that is fixed (α) and the taut spring initial length (μ) where $\mu < 1-\alpha$. Equating ω_x and ω_θ from (14) yields the following relationships between μ and α.

$$\mu^* = (1-\alpha)(1-\frac{\alpha}{3}) \quad , \quad \alpha^* = 2[1-\sqrt{1-\frac{3}{4}(1-\mu)}\,] \quad (15)$$

Consequently, the existence of each case is as follows:

i) $\omega_y < \omega_x < \omega_\theta \quad ; \quad \mu < \mu^*$

ii) $\omega_y < \omega_x = \omega_\theta \quad ; \quad \mu = \mu^*$

iii) $\omega_y < \omega_\theta < \omega_x \quad ; \quad \mu^* < \mu < (1-\alpha)$

We note that the upper bound, $\mu = (1-\alpha)$, yields $\omega_y = \omega_\theta = 0$, which consists of the strongly coupled system with no linear stiffness in y or θ. Furthermore, the parameter space defined by case (iii) is much smaller than that of case (i).
The natural frequencies in (14) reveal the possible existence of internal resonances described by $\ell\omega_i = m\omega_j = n\omega_k$ where ℓ, m, n are integers. We limit our analysis to $\ell = 1$ and $n > m$, and obtain the following conditions for internal resonances.

i) $\omega_y < \omega_x < \omega_\theta \quad : \quad \omega_\theta = m\omega_x = n\omega_y$

$$\alpha = (\frac{3}{n^2}) \quad , \quad \mu = (1-\frac{3}{n^2})[1-(\frac{m}{n})^2] \quad ; \quad m > 1, n > 2, n > m \quad (16)$$

Examples of internal resonances from (16) are (1:m:n) = {(1:2:3), (1:2:4), (1:2:5), (1:3:4), (1:3:5), (1:4:5)} where the resulting elements length $\alpha \le 1/3$.

ii) $\omega_y < \omega_x = \omega_\theta$: $\omega_\theta = \omega_x = n\omega_y$

$$\alpha = (\frac{3}{n^2}) \quad ; \quad \mu = (1 - \frac{3}{n^2})(1 - \frac{1}{n^2}) \quad ; \quad n > 1 \tag{17}$$

This case admits all possible (1:n) internal resonances. However, an element length of $\alpha \le 1/3$ precludes (1:2).

iii) $\omega_y < \omega_\theta < \omega_x$: $\omega_x = m\omega_\theta = n\omega_y$

$$\alpha = 3(\frac{m}{n})^2 \quad , \quad \mu = [1 - 3(\frac{m}{n})^2]\,(1 - \frac{1}{n^2}) \quad ; \quad m > 1\,, (\frac{m}{n})^2 < \frac{1}{3}, n > m \tag{18}$$

Examples of internal resonances from (18) are (1:m:n) = {(1:2:4), (1:2:5), (1:2:6), (1:3:6)}. However, only (1:2:6) yields an element length of $\alpha = 1/3$ whereas the other combinations result in larger lengths.

4. FORCED VIBRATION WITHOUT LOCALIZED ELEMENT ACTUATION

We focus in this paper on two fundamental cases that satisfy the conditions for internal resonance described in the previous section: i) small (nondimensional) element size ($\alpha = 0.12$), and ii) a finite size element corresponding to a third of the span ($\alpha = 1/3$). We present some reference results for the uncontrolled (e.g. without element actuation: $\beta = 0$). The candidate cases selected were investigated for small (nondimensional) exciting amplitudes ($\gamma = 0.01 - 0.1$), and small damping determined from a critical damping ratio describing underdamped conditions ($\zeta = 0.001 - 0.01$). The forcing frequency selected was close to the horizontal natural frequency ω_x ($\Omega_1 = 1.5$).
Attention is focused on two conditions of internal resonance of type (ii). Figure 1 depicts subharmonic state space projections at steady state (y'(y) where y=x[3], y'=x[4]) and its respective Poincare' map (denoted by Xp[3], Xp[4]), which includes three points stroboscopically sampled at the fundamental exciting period (e.g. $T = 2\pi / \Omega_1$). Figure 1a ($\zeta = 0.01$) describes the third subharmonic dynamics of a taut element where the element length and stretch satisfy the internal resonance of $\omega_x = \omega_\theta = 3\omega_y$ deduced from (17) ($\alpha = 1/3, \mu^* = 0.5926$). Figure 1b ($\zeta = 0.018$) describes similar subharmonic dynamics of the smaller element with less stretch ($\alpha = 0.12, \mu = 0.8448$). Recall that the degree of nonlinearity is controlled by the stretch (or the closeness of the initial length to midspan). Thus, the strongest nonlinearity is obtained for $\mu = 1 - \alpha$ (or $\ell_0 = b - a_0$) and the degree of nonlinearity is portrayed by a larger volume of state space. We also consider here an additional case (not shown) of the internal resonance of type (iii) where an element with the stretch deduced from (18) satisfies the relationship $\omega_x = 2\omega_\theta = 6\omega_y$ ($\alpha = 1/3, \mu = 0.666$). This case corresponds to the nearly zero pretension as $\mu \sim 1 - \alpha$.
We note that the forced vertical vibration of the system without element actuation includes solutions which consist of simple, small amplitude, periodic response that coexist with the large variety of ultrasubharmonics predicted by the internal resonances of the previous section.

5. ACTUATION OF THE LOCALIZED ELEMENT

Actuation of the smart element is obtained by harmonic perturbation of its length (9) with an amplitude that is smaller than half the element length ($\alpha > \beta$) and a frequency of vibration (Ω_2) that can coincide or is incommensurate with that of the forcing frequency (Ω_1). Results from a numerical bifurcation analysis are obtained and discussed to reveal possible reduction of the directly excited vertical vibration via: i) parametric excitation, and ii) energy transfer from the directly excited vertical response to both rotation and horizontal motions.

Figure 2 describes bifurcation diagrams (calculated using the reduced system (13) and then verified with (7)) that depict the response versus Ω_2 of both small and finite elements described above and excited near the horizontal natural frequency ($\gamma = 0.05, \Omega_1 = 1.5$) with small (nondimensional) actuation ($\beta = 0.01$). The vertical response of the small element ($\alpha = 0.12, \mu = 0.8448$) in Figure 2a reveals existence of both aperiodic dynamics (e.g. quasiperiodic/chaotic) and periodic response for mode-locked frequencies (e.g., $\Omega_2 = m\Omega_1 / n$). Examples of both periodic and quasiperiodic response are portrayed in Figure 3. Note that the quasiperiodic response (Figure 3a: $\Omega_2 = 0.7$) is defined by a non finite number of Poincare' points that describe an invariant topology whereas the periodic dynamics is depicted by a single point (Figure 3b: $\Omega_2 = 3.0$). The reduction in vibration amplitude is apparent for both examples (compare Figure 3 with Figure 1b) and is pronounced for vibrations with a large frequency ($\Omega_2 > 2.5$).
The vertical response of the larger element ($\alpha = 1/3, \mu = 0.666$) in Figure 2b is aperiodic (for the range of frequencies tested) and reduces the directly excited vibration for parameters governing internal resonance conditions where the mechanism for vibration reduction consists of energy transfer to rotation and slight horizontal translation. This can be observed from the quasiperiodic angular state space ($\theta = x[5], \theta' = x[6]$) and Poincare' map (Xp[5], Xp[6]) in Figure 4b ($|\theta| \sim 0.28 rad$). Note that the strong nonlinearity of this case does not yield (for the given damping) the anticipated periodic mode-locked response for element actuation at the period of the direct excitation ($\Omega_2 = \Omega_1 = 1.5$).

6. CLOSING REMARKS

We have derived a lumped mass model for a taut string dynamical system that incorporates a smart localized piezoelectric element that can change its length to counteract the externally excited vibrations in the vertical plane. The strategy employed consists of an open loop multifrequency excitation approach enabled via harmonic actuation of the smart element by a slight harmonic change of its length. We have demonstrated the possible reduction of vertical vibrations due to external excitation in the vertical plane.
Result of a numerical bifurcation analysis reveal possible vibration reduction via two distinct mechanisms: i) parametric excitation, and ii) energy transfer from the directly excited vertical response to both rotation and horizontal motions. We note that the former appears more robust than manipulation of internal resonances.
Future research will explore more systematically the influence of smart element actuation on system dynamics. Furthermore, it will concentrate on a finite set of localized structural elements (the minimal number and optimization of their location) for a variety of environmental conditions. A parallel effort should be devoted to a continuum description for the piezoelectric sleeves and identification of their properties from a controlled set of experiments.

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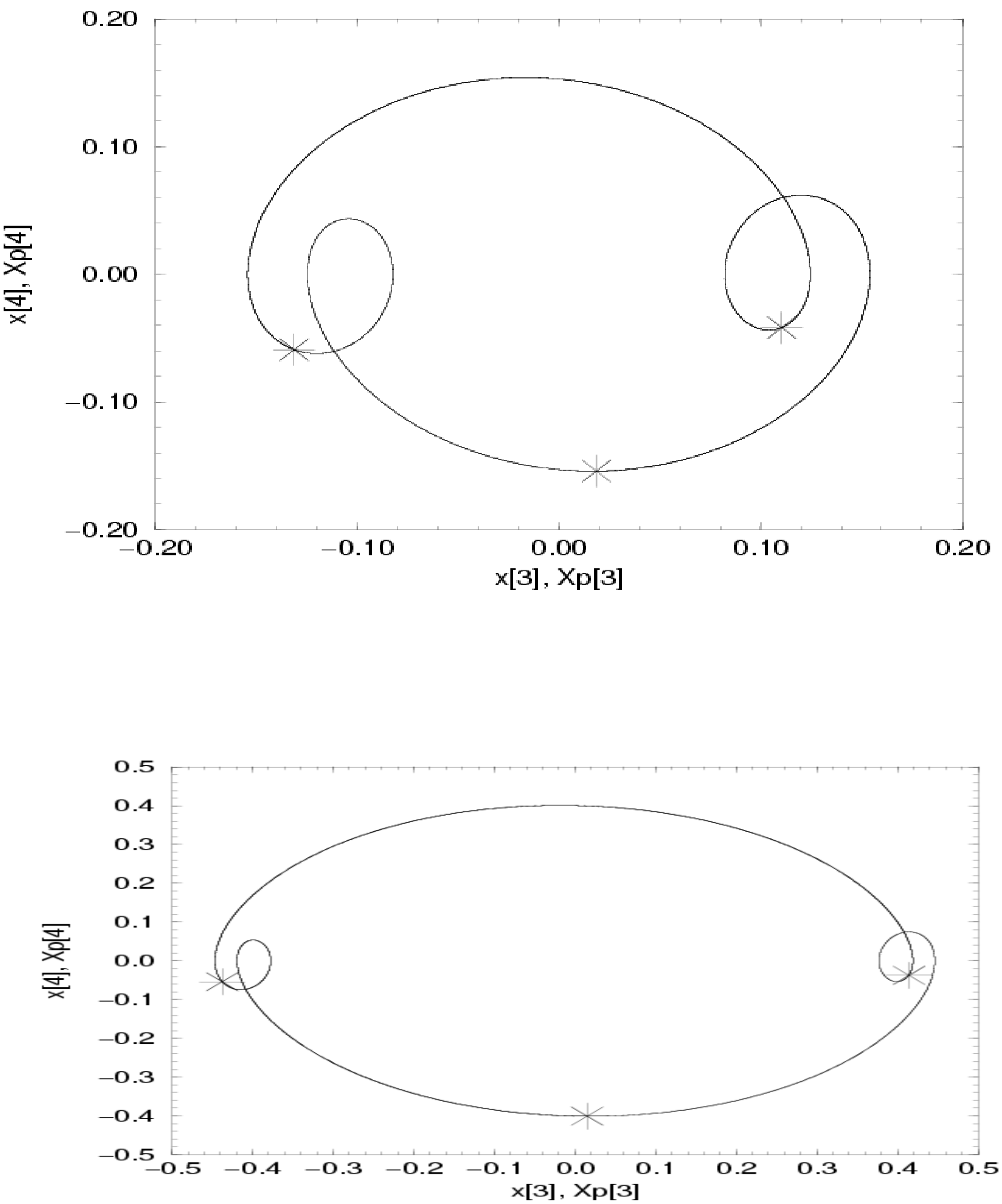


Fig 1 Third subharmonic response without element actuation ($\Omega_1 = 1.5$): state space (dy/dt(y), y=x[3], dy/dt=x[4]) and Poincare' map (Xp[4](Xp[3])) for a) $\alpha = 1/3, \mu = 0.5926$, b) $\alpha = 0.12, \mu = 0.8448$.

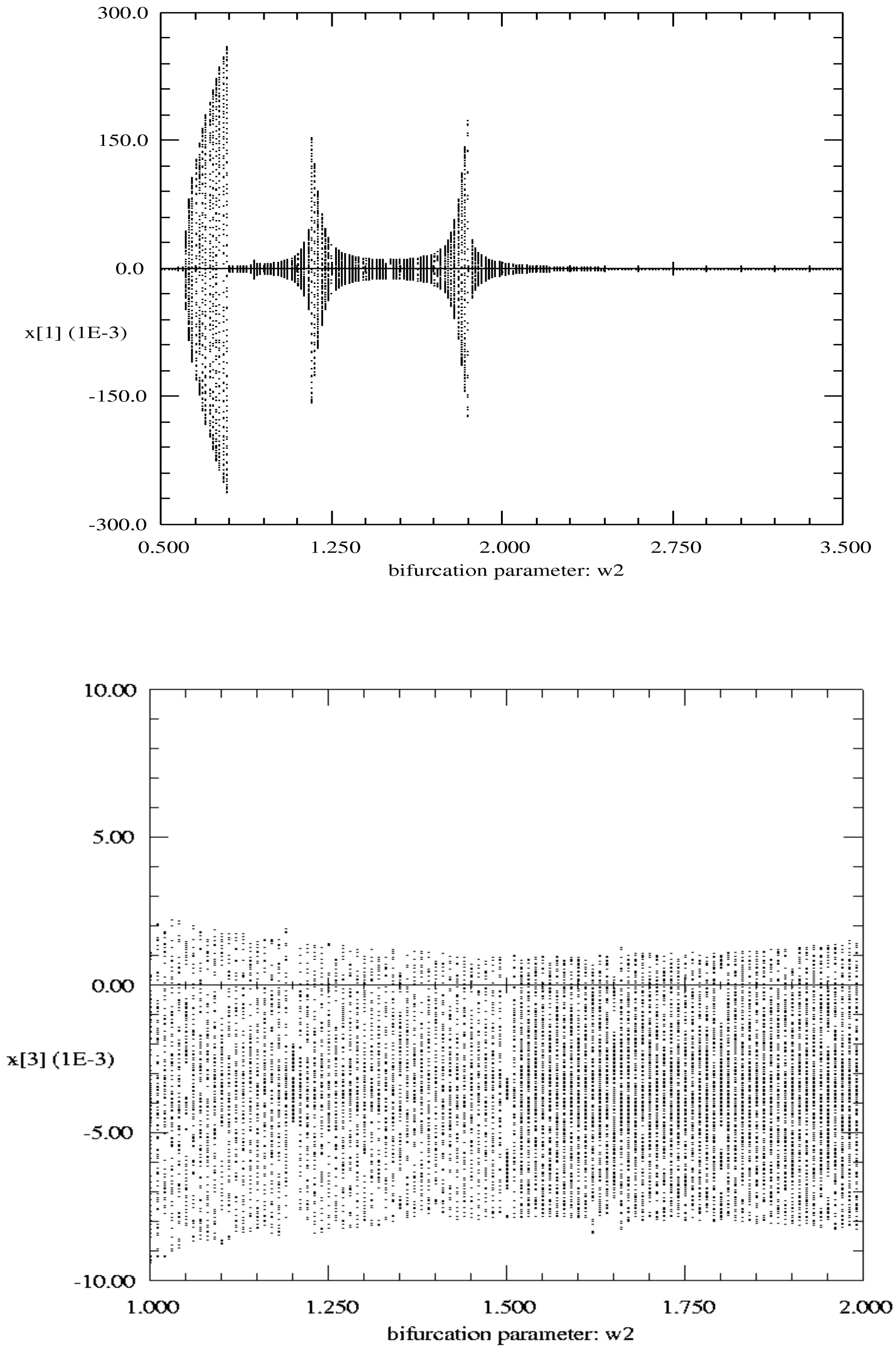
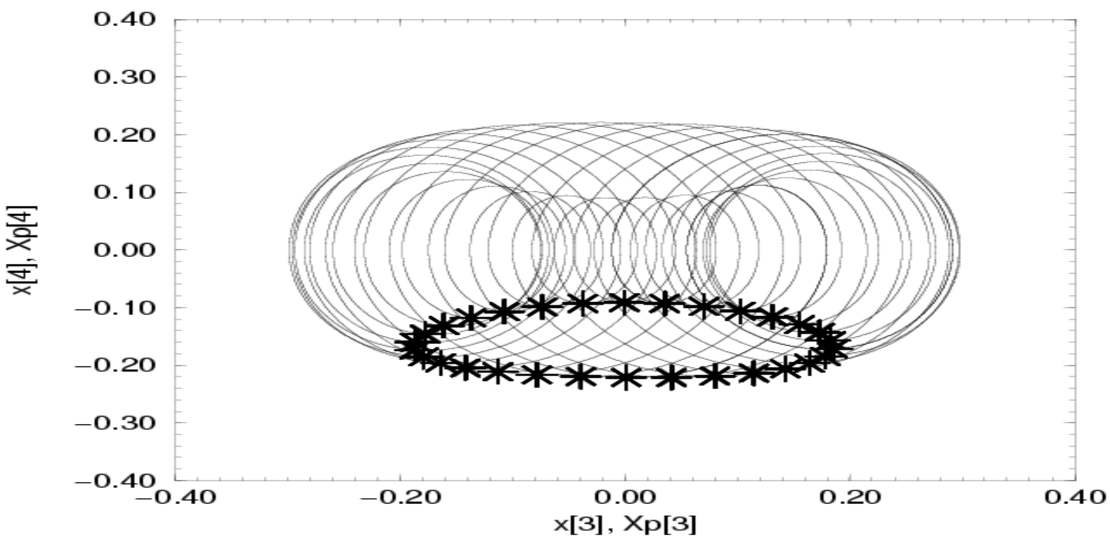


Fig.2 Bifurcation diagrams (y(w2= Ω_2), $\beta = 0.01, \gamma = 0.05, \Omega_1 = 1.5$) for: a) $\alpha = 0.12, \mu = 0.8448$, b) $\alpha = 1/3, \mu = 0.666$.



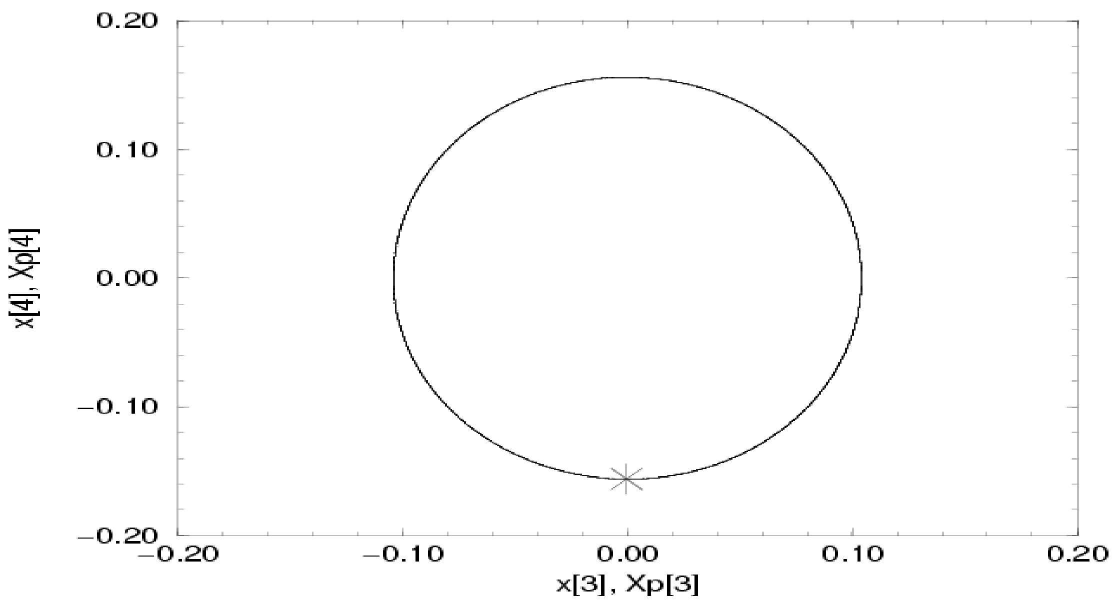
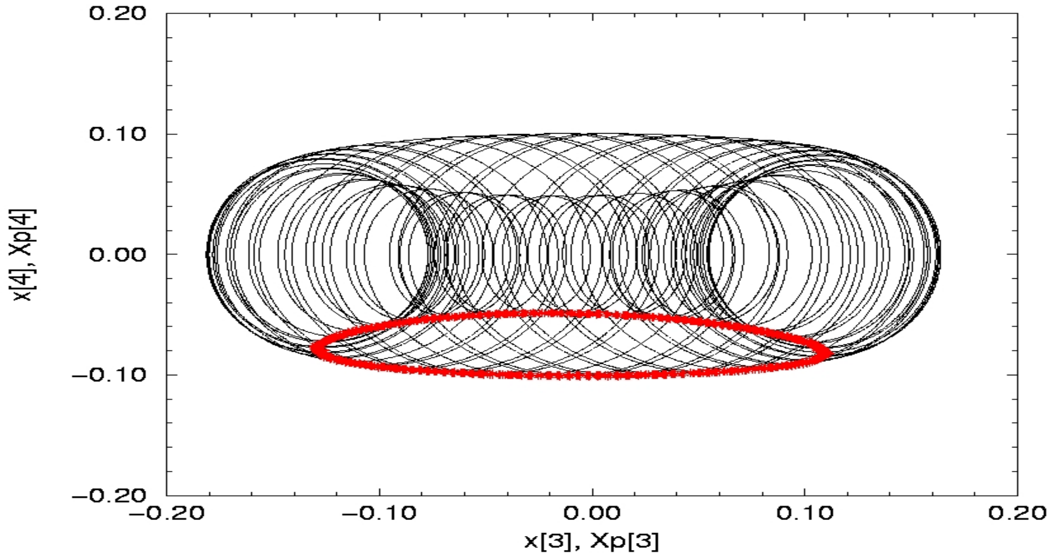


Fig 3 Modulated response of the actuated element ($\alpha = 0.12, \mu = 0.8448, \beta = 0.01, \gamma = 0.05, \Omega_1 = 1.5$): state space (dy/dt(y), y=x[3], dy/dt=x[4]) and Poincare' map (Xp[4](Xp[3])) for a) $\Omega_2 = 0.7$, b) $\Omega_2 = 3.0$.



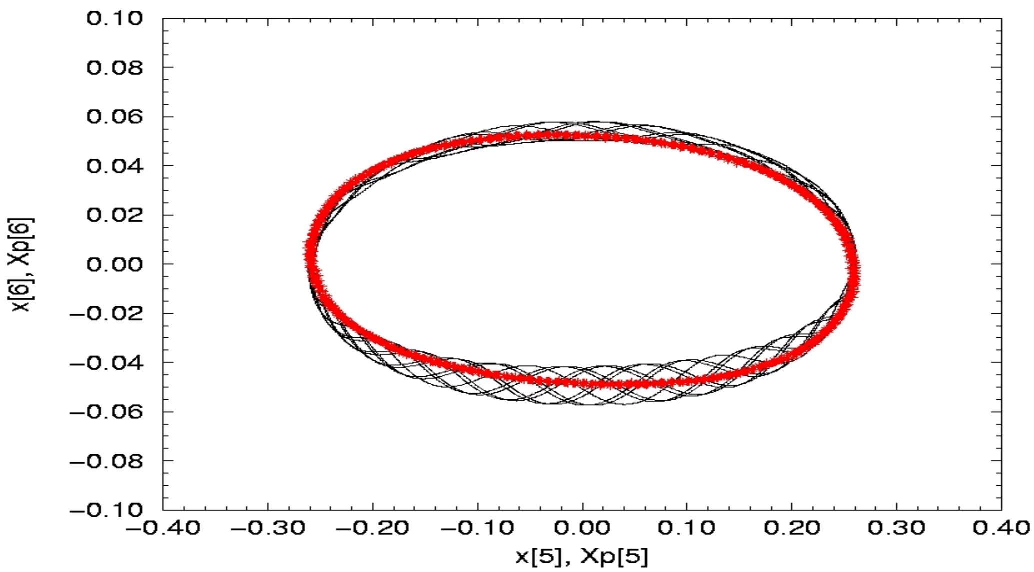


Fig 4 Modulated response of the actuated element ($\alpha = 1/3, \mu = 0.666, \beta = 0.01, \gamma = 0.05, \Omega_1 = \Omega_2 = 1.5$): a) state space (dy/dt(y), y=x[3], dy/dt=x[4]) and Poincare' map (Xp[4](Xp[3])) , b) state space ($d\theta / dt(\theta), \theta = x[5], d\theta / dt = x[6]$) and Poincare' map (Xp[6](Xp[5])).