The Bifurcation Structure of a Self-Excited Inertia Wheel Double Pendulum

Y. Levi and O. Gottlieb

Department of Mechanical Engineering, Technion-Israel Institute of Technology, Haifa, Israel.

Summary. We construct the bifurcation structure of a self-excited inertia wheel double pendulum. The dynamical system exhibits stable equilibria, periodic limit cycle oscillations and nonstationary rotations. We investigate several configurations documented in literature which exhibit internal resonances and compare their bifurcation structure demonstrating similarities in periodic oscillations and distinct differences in patterns of chaotic rotations.

Introduction

Self-excitation of restrained and freely oscillating rigid bodies are encountered in a variety of engineering applications including friction induced vibration, aeroelastic limit cycle oscillations and fluid-structure interaction. Stabilization of limit cycle oscillations has been proposed by several approaches including boundary feed forward control of a multi-tethered lighter-than-air sphere [1] and digital delayed feedback control of an aero-pendulum [2]. Of particular importance is the capability of an inertia wheel (or reaction wheel) with linear and nonlinear feedback to obtain stable and robust limit-cycle oscillations in underactuated dynamical systems [3,4]. We thus investigate the self-excited dynamics of an autonomous dynamical system which consists of a planar double pendulum augmented with a rotating inertia wheel (see Fig.1-left). We consider double pendulum configurations documented in literature which exhibit conditions near a 1:1 [5], 2:1 [6] and 3:1 [7] internal resonances. Stability analysis of the zero-angle equilibrium position under a partial-state feedback scheme yields a stability map of feedback gains (Fig.1-right) that includes a distinct region (green) of possible self-excited limit cycles that is bounded by Hopf and Saddle-Node bifurcations denoting the transitions from stable equilibria to periodic limit-cycle oscillations (solid blue line) and the transition to rotations (dashed red line), respectively.

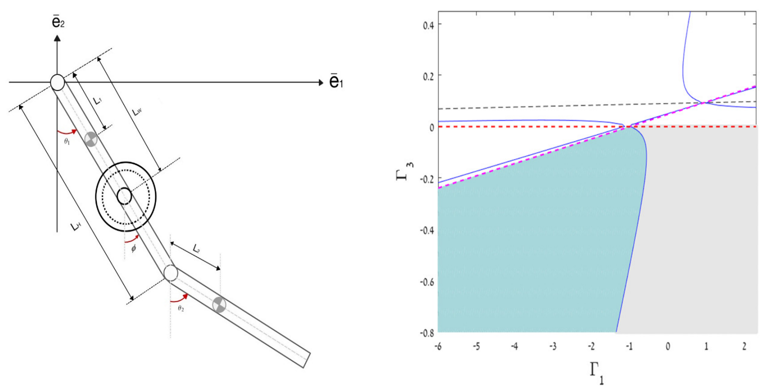

Figure 1. Definition sketch (left) and stability map of the inertia wheel feedback gains for a 2:1 configuration depicting regions of stable zero equilibrium (grey), possible limit-cycles (green) and rotations (white).

Results

We perform a numerical investigation of the dynamical system near a 1:1 internal resonance where the states include the angles (X1, X3) and angular velocities of the pendula and wheel (X2, X4, X5), respectively. The bifurcation diagram (Fig.2-left) depicts the maximal state oscillation amplitude as a function of the wheel velocity feedback gain (Γ_3) and includes two regions of periodic oscillations (about the stable zero equilibrium and about the unstable upper equilibrium) and two regions of nonstationary oscillations and rotations. Examples of a periodic limit cycle (Fig.2-center) and a chaotic rotation (Fig.2-right) include state space projections of the system conjugate momenta (P_i, i=1,2,3) and the bottom pendulum position (Y(X) overlaid with their corresponding Poincare' maps (sampled every positive zero crossing of the bounded wheel velocity).

Discussion

We numerically integrate the dynamical system using the conditions of the above noted 2:1 and 3:1 internal resonances and normalize their bifurcation structures by their respective Hopf thresholds (Γ_3/Γ_{3Hopf}). We portray the normalized bifurcation strucutres (Fig.3-left) demonstrating a distinct similarity of the periodic self-excited limit cycles between the configurations of the 1:1 (blue), 2:1 (red) and 3:1 (green) internal resonances.) However, the Poincare' maps of the nonstationary rotations for the 2:1 (Fig.3-center) and 3:1 (Fig.3-right) configurations reveal a distinctly different chaotic pattern than the one obtained for the configuration near a 1:1 internal resonance (Fig.2-right). Furthermore, while all the considered cases reveal similar behavior of the inertia wheel velocity (X5), the velocities of the two pendulum elements (X2, X4) exhibited a distinctly different behavior between the region of periodic limit cycle oscillations and nonstationary rotations.

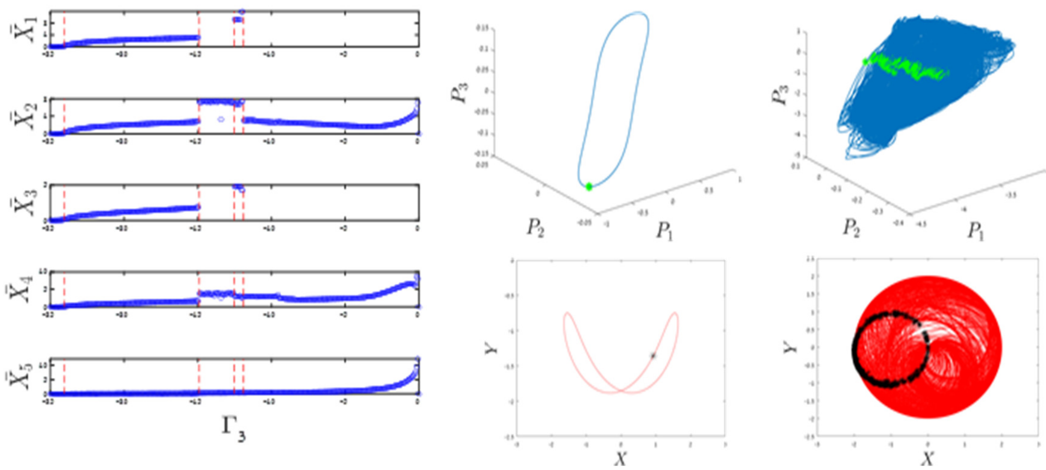

Figure 2. Bifurcation diagram (left), and overlaid state space with Poincare' map for periodic limit cycle oscillations (center) and nonstationary rotations (right).

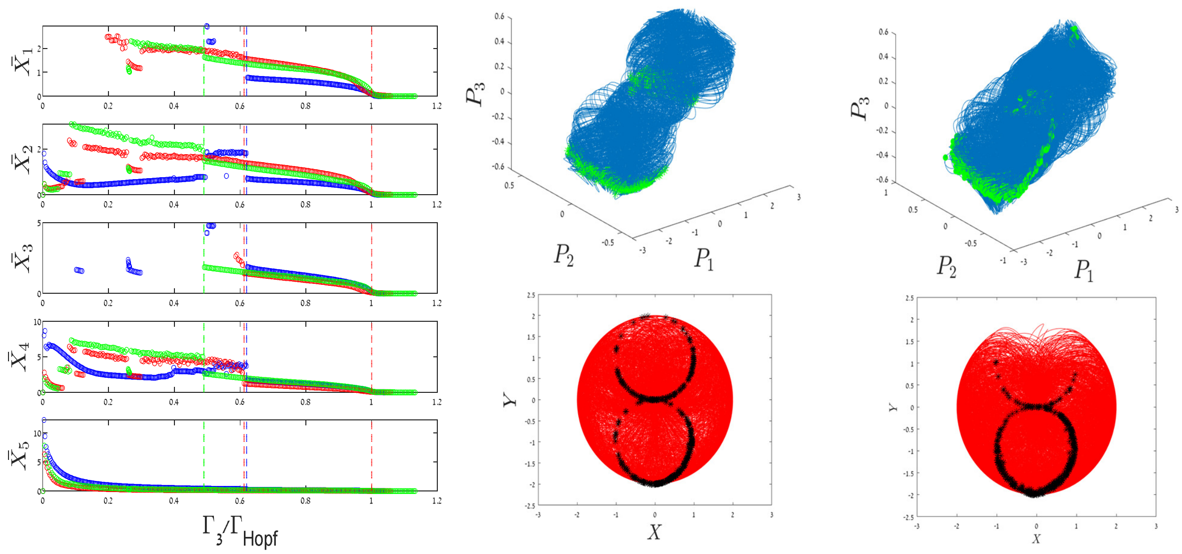

Figure 3. Normalized bifurcation diagram (left) depicting the configurations of the 1:1 (blue), 2:1 (red) and 3:1 (green) internal resonances and overlaid state space and Poincare' map for nonstationary rotations of the 2:1 (center) and 3:1 (right) internal resonances.

References

[1] Mi L. and Gottlieb O., Nonlinear Dynamics, 93: 1353–1369, 2018.
[2] Habib G., Miklos A., Enikov E.T., Stepan G., and Rega G., In. J. Dynamics and Control, 5: 629-643, 2017.
[3] Alonso D.M., Paolini E.E., and Moiola J.L., Nonlinear Dynamics, 40: 205–225, 2005
[4] Haddad, N.K., Belghith, S, Gritli, H. and Chemori, A. Int. J. Bifurcation and Chaos, 27: 1750104, 2017.
[5] Dudkowsky D., Wojewoda J., Czolczynski K. and Kapitaniak T., Nonlinear Dynamics, 102: 759–770, 2020.
[6] Levien, R.B, Tan S.M, American J. Physics, 61: 1038-1044, 1993.
[7] Vadai G., Gingl Z., Mellar J., European J. Physics, 33: 907-920, 2012.