# Chimera States in Elastically Coupled Self-Excited Underactuated Rigid-Body Arrays

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**Abstract**. We investigate the bifurcation structure and the onset of chimera states in coupled self-excited underactuated rigid-body arrays. The dynamical systems considered consist of two elastically coupled arrays of identical single and double pendula augmented by inertia wheels which exhibit asymptotically stable equilibria, periodic limit cycle oscillations, and non-stationary rotations. The analysis reveals that synchronous periodic oscillators are in-phase whereas quasiperiodic oscillators are out-of-phase. Furthermore, non-stationary solutions exhibit combinations of quasiperiodic and chaotic oscillations and rotations of asynchronous individual elements culminating with coexisting synchronous and chimera states.

## 1. Introduction and Problem Formulation

Self-excited synchronous oscillations in multibody dynamical systems have been documented since the middle of the seventeenth century. Huygens made the amazing observation that two pendulum clocks hanging from a common flexible support swung together periodically approaching and receding in opposite motions [1]. During the last two decades there has been a growing interest in the stability and robustness of continuous and intermittent synchronization of periodic and nonstationary oscillations which in addition to neural network populations have been observed in nanomechanical resonator arrays [2] and in experiments of mechanical networks [3]. Of particular interest are the chimera states in which the symmetry of an oscillator population is broken into a synchronous part and an asynchronous part culminating with a novel class of decoherent behaviour [4]. In this research we investigate the bifurcation structure and the emergence of chimera states in a pair of elastically coupled pendulum arrays that are augmented by inertia wheels governed by a linear feedback mechanism. Figure 1 depicts the configurations of a single (Fig.1 left) and double (Fig.1 right) arrays. We derive the equations of motion and examine the complexity of coexisting synchronous and asynchronous self-excited oscillations in the coupled arrays.

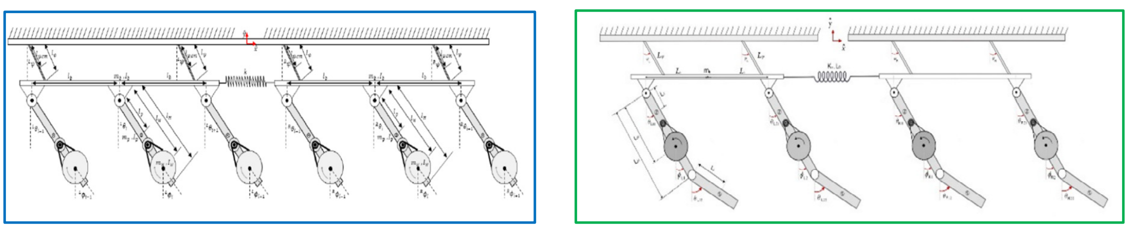

Figure 1: Sketch of the elastically coupled single (left) and double (right) inertia wheel pendulum arrays.

## 2. Discussion of Results

We combine an analytical and numerical investigation to determine the bifurcation structure of the self-excited elastically coupled arrays which exhibit periodic limit cycle oscillations and non-stationary rotations. Figure 2 depicts the bifurcation structures of the coupled arrays as a function of the nondimensional stiffness coupling parameter. We note that both the single (Fig.2 left) and double (Fig.2 right) pendulum configurations exhibit a distinct range of stiffness where limit cycle oscillations occur (marked as region II). However, only the single pendulum array includes a region of nonstationary oscillations (marked as III).

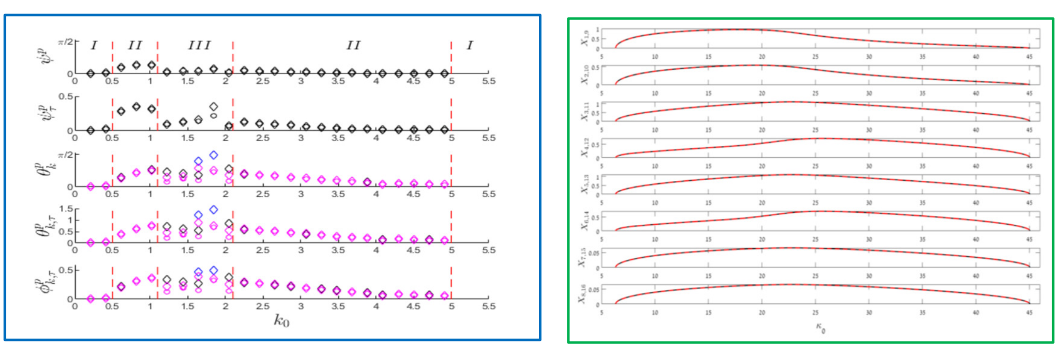

Figure 2: Bifurcation diagrams of the elastically coupled single (left) and double (right) arrays.

We also investigate the synchronous dynamics and the emergence of chimera states within the system and make use of the Kuramoto order parameter [4]. This parameter enables identification of synchronized in-phase or anti-phase solutions where the order parameter for both arrays is unity whereas a chimera state is portrayed when the order parameter for one array is unity and the order parameter of the second array varies in an irregular manner between zero (describing a decoherent state) and unity (a synchronous state). Figure 3 depicts example chimera states of the coupled single inertia wheel pendulum array as a function of increasing coupling stiffness parameter. We note that a small coupling stiffness reveals a transition from an IP synchronized state of periodic limit-cycles (Fig.3 left) to a chimera state (Fig.3 centre-left) where the quasiperiodic response of one array (blue) is synchronized whereas the second array (red) is decoherent. A further increase of the stiffness (Fg.3 centre-right) reveals that both of the coupled arrays are decoherent (both red and blue arrays are not synchronized) culminating with IP synchronization of both coupled arrays for large stiffness.

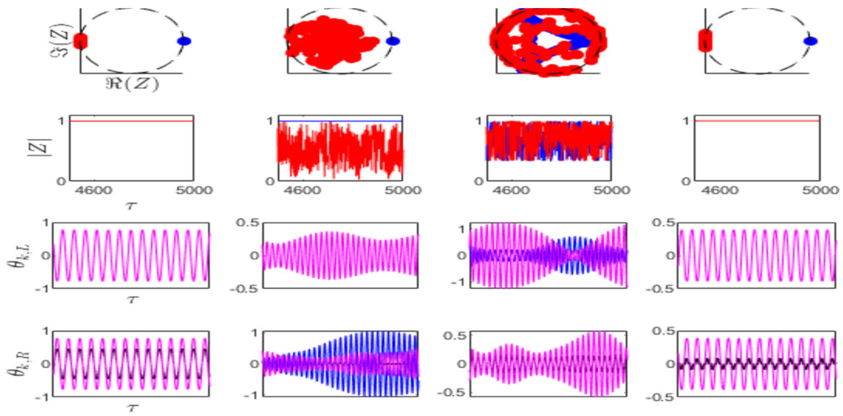


Figure 3: Chimera states of the elastically coupled single pendulum arrays.

Figure 4 depicts a transition from a synchronized state of periodic limit-cycle solutions (Fig.4 left) to a chimera state of chaotic rotations (Fig.4 center) culminating with a decoherent state of chaotic oscillations (Fig.4 right) of the double pendulum arrays.

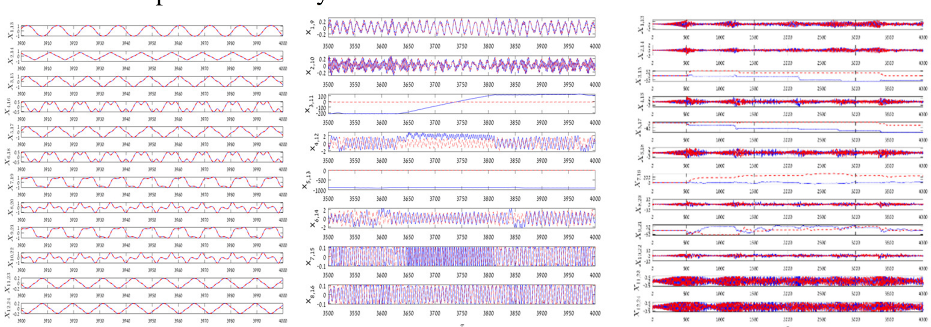

Figure 4: Synchronized (left), chimera (center) and decoherent (right) states of the double pendulum arrays.

The combined analytical and numerical methodologies employed enable construction of a comprehensive bifurcation structure that sheds light on emergence of chimera states that appear at the transition from synchronized oscillations to decoherent rotations in elastically coupled single and double inertia wheel pendulum arrays.

## References

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