Asymptotic model-based estimation of nonlinear viscoelastic damping in magnetomotive nanowires

Chen Shmulman, Evyatar Hacker and Oded Gottlieb
Department of Mechanical Engineering, Technion-Israel Institute of Technology, Israel

Summary. We identify the existence of nonlinear viscoelastic damping in conductive metallic nanowires that are subject to magnetic excitation and investigate its effects on the spatiotemporal dynamical system response. We derive a consistent continuum-based modal dynamical system for a geometrically nonlinear viscoelastic nanowire that is subject to magnetic excitation and employ a combined asymptotic and numerical methodology to estimate the magnitude of viscoelastic damping from controlled benchmark nanowire experiments. The criteria for bistable planar response estimated from experiments enables estimation of the cubic viscoelastic damping parameter for small magnetomotive excitation culminating with the transition to three-dimensional periodic and nonstationary whirling nanowire dynamics with increasing magnitude of excitation.

The significance of nonlinear damping has been documented for both micro- and nano electromechanical systems [1-4] where the magnitude of the effective quality factor measured in controlled experiments has been shown to decrease significantly with increasing amplitude of excitation [4: Fig.2 inset]. Furthermore, while the transition from planar response to three-dimensional whirling dynamics has been investigated systematically for large scale models [5 and references within], conditions for spatio-temporal whirling in micro/nano-scale models have yet to be determined. We thus formulate a continuum-based geometrically nonlinear three-dimensional initial boundary-value problem (IBVP) for a highly-tensioned nanowire (see Figure 1 left) which consistently incorporates cubic viscoelastic damping assuming a Voigt-Kelvin constitutive law [1,2,5] and magnetic excitation [6]. The IBVP is then reduced to a modal dynamical system describing three-dimensional whirling that exhibit a 1:1 internal resonance between the in-plane and out-of-plane oscillation directions. We employ a generalized averaging formulation to obtain the reduced-order system slowly varying evolution equations. Analysis of the in-plane amplitude evolution equation [7] yields the system damping backbone curve (Figure 1 right) which enables comparison with measured data from controlled experiments. The intersection of the zero-response amplitude with the effective equivalent damping parameter yields the magnitude of the linear system damping parameter and the slope of the backbone curve enables estimation of the nonlinear viscoelastic parameter as a function of increasing magnetic excitation.

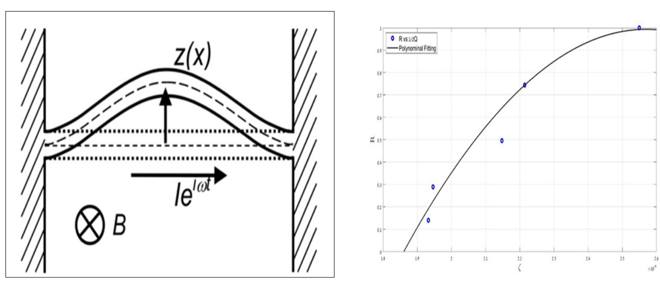

Figure 1. Magnetomotive nanowire sketch (left) and damping backbone curves from experimental data in [2] (right).

The slowly varying evolution equations for the planar case are investigated to yield the following: (i) an increase of cubic damping with constant magnetic excitation (Figure 2 left) culminates with a decrease of the maximal amplitude and consequent elimination of bistable solutions (ii) an increase of cubic damping with a corresponding increase of magnetic excitation enables maintain an equal hysteresis frequency range.

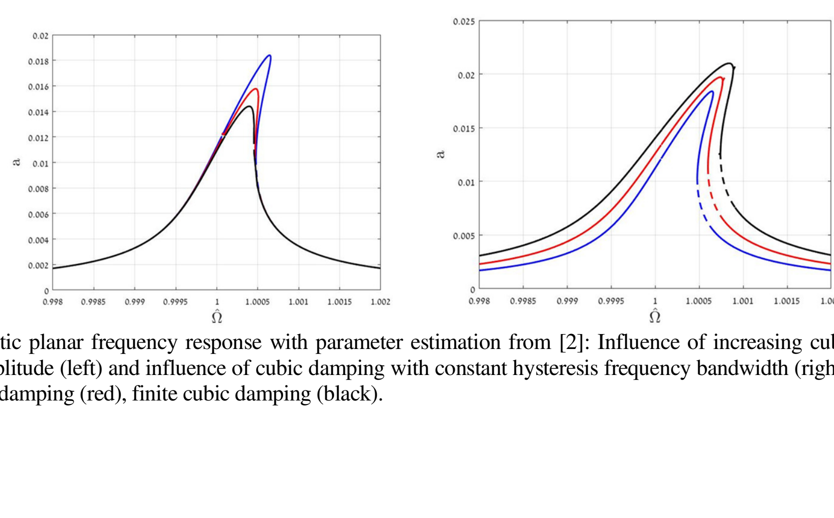

Figure 2. Asymptotic planar frequency response with parameter estimation from [2]: Influence of increasing cubic damping with fixed magnetic input amplitude (left) and influence of cubic damping with constant hysteresis frequency bandwidth (right) - zero cubic damping (blue), small cubic damping (red), finite cubic damping (black).

Analysis of the coupled slowly-varying evolution equations yield criteria for two thresholds describing: (i) the transition from planar periodic oscillations with finite linear damping to three-dimensional periodic whirling dynamics and (ii) the transition from periodic whirling to nonstationary three-dimensional oscillations which can be quasiperiodic or chaotic [5]. Numerical integration of the dynamical system for different values of system parameters near its one-to-one internal resonance reveal periodic (Figure 3 left), quasiperiodic (not shown) and chaotic whirling (Figure 3 right) where the direction of angular momentum is sensitive to initial conditions. We note that in this analysis we have ignored the influence of boundary damping and thermoelastic damping that are expected to govern dissipation mechanisms for nanowires operating at very low temperature [4].

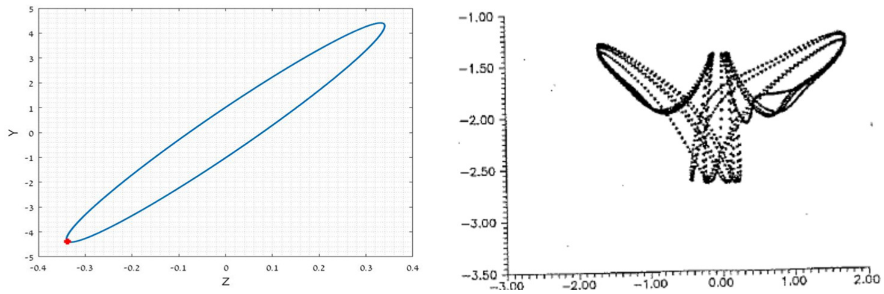

Figure 3. Numerical response for spatio-temporal whirling dynamics: periodic state-space projection (left) and chaotic Poincare' map (right).

References

[1] Zaitsev S., Shtempluck O., Buks E. and Gottlieb O., Nonlinear damping in a micromechanical oscillator. Nonlinear Dynamics 67: 859-883 (2012).

[2] Mora K. and Gottlieb O., Parametric excitation of a micro-beam string with asymmetric electrodes: multimode dynamics and the effect of nonlinear damping. Journal of Vibration and Acoustics 139: 040903, 1-9 (2017).

[3] Erbe A., Krommer H., Kraus A. and Blick R.H., Mechanical mixing in nonlinear nanomechanical resonators. Applied Physics Letters, 77, (19): 3102-3104 (2000).

[4] Husain A., Hone J., Postma H.W., Huang M.H., Drake T., Barbic M., Scherer A. and Roukes M.L., Nanowire-based very-high-requency electromechanical resonator, Applied Physics Letters 83 (6): 1240-1242 (2003).

[5] Leamy M.J. and Gottlieb O., Internal resonances in whirling strings involving longitudinal dynamics and material non-linearities. Journal of Sound and Vibration 236 (4): 683-703 (2000).

[6] Hacker E. and Gottlieb O., Local and global bifurcations in magnetic resonance force microscopy. Nonlinear Dynamics, 99: 201-225 (2020).

[7] Gottlieb O. and Habib G., Nonlinear model-based estimation of quadratic and cubic damping mechanisms governing the dynamics of a chaotic spherical pendulum. Journal of Vibration and Control 18 (4): 536-547 (2011).