Bifurcations and Chimera States in Self-Excited Inertia Wheel Pendulum Arrays

Gilad Yakir[*], Yuval Levi[*] and Oded Gottlieb[*]

[*]*Department of Mechanical Engineering, Technion-Israel institute of Technology, Haifa, Israel*

Abstract. We investigate the bifurcation structure and the onset of chimera states in a coupled pair of self-excited inertia wheel pendula arrays. The dynamical system exhibits asymptotically stable equilibria, periodic limit cycle oscillations, and non-stationary rotations. The analysis reveals that synchronous periodic oscillators are in-phase whereas quasiperiodic oscillators are out-of-phase. Furthermore, non-stationary rotations exhibit combinations of oscillations and rotations of the individual elements which are asynchronous culminating with coexisting synchronous and chimera-like solutions.

Introduction and Problem Formulation

Self-excited synchronous oscillations in multibody dynamical systems have been documented since the middle of the seventeenth century. Huygens made the amazing observation that two pendulum clocks hanging from a common flexible support swung together periodically approaching and receding in opposite motions [1]. During the last two decades there has been a growing interest in the stability and robustness of continuous and intermittent synchronization of periodic and nonstationary oscillations which in addition to neural network populations have been observed in nanomechanical resonator arrays [2] and in experiments of mechanical networks [3]. Of particular interest are the chimera states in which the symmetry of an oscillator population is broken into a synchronous part and an asynchronous part culminating with a novel class of decoherent behaviour [4]. In this research we investigate the emergence of bifurcations and chimera states in a pair of elastically coupled self-excited inertia wheel double pendulum arrays depicted in Figure 1 (left). We derive the equations of motion and examine the complexity of coexisting synchronous and asynchronous self-excited oscillations in the coupled arrays each with three planar pendula augmented with rotating inertia wheels governed by a linear feedback mechanism.

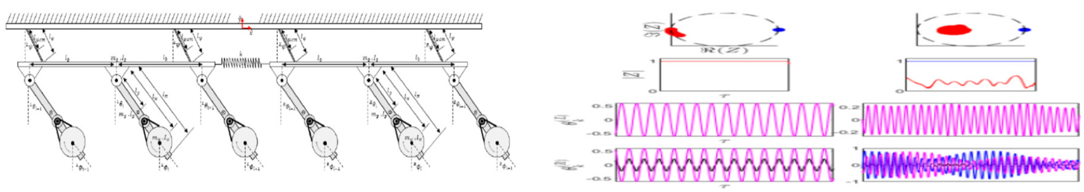

Figure 1: Definition sketch of the coupled pair of inertia wheel pendulum arrays (left) and example in-phase synchronized response and nonstationary decoherent chimera oscillation dynamics (right).

Results and Discussion

We combine an analytical and numerical investigation to determine the bifurcation structure of the self-excited elastically coupled arrays which exhibit periodic limit cycle oscillations and non-stationary rotations. We investigate the synchronous dynamics and the emergence of chimera states within the system and make use of the Kuramoto order parameter [4] which enables identification of synchronized in-phase or anti-phase solutions where the order parameter for both arrays is unity in comparison to chimera state where the order parameter for one of the coupled arrays varies in an irregular manner between zero (describing a decoherent state) and unity (a synchronous state) as shown in Figure 1 (right). The combined analytical and numerical methodologies employed enable construction of a comprehensive bifurcation structure that sheds light on emergence of chimera states, synchronization and decoherence in the elastically coupled arrays in both oscillation and rotation regimes.

References

[1] Ramirez J.P., Olvera L.A., Nijmeijer H., Alvarez J. (2016). The sympathy of two pendulum clocks: beyond Huygens' observations. Scientific Reports **6**: 23580.

[2] Kambali, P., Torres, F., Barniol, N., Gottlieb, O. (2019). Nonlinear multi-element interactions in an elastically coupled microcantilever array subject to electrodynamic excitation. Nonlinear Dynamics **98**: 3067-3094.

[3] Kapitniak T., Kuzma P., Wojewoda J., Czolczynski K, Maistrenko Y. (2014). Imperfect chimera states for coupled pendula. Scientific Reports **4**:1-4.

[4] Martens E., Thutupallil S., Fourrière1 A., Hallatschek O. (2013). Chimera states in mechanical oscillator networks. Proc. National Academy of Sciences, **110**:10563-10567.